II

Demand Deposits, Trading Restrictions, and Risk Sharing

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This paper explores the role of demand deposits in risk sharing. Demand deposits coupled with restricted trading opportunities are shown to provide greater risk sharing than equity shares that are traded freely. Trading restrictions allow the achievement of allocations using demand deposits superior to those obtained using equity shares—with or without trading restrictions. The removal of the trading restrictions, however, eliminates the advantage of demand deposits. In this case, demand deposits and equity shares are shown to provide identical risk-sharing opportunities.

1. Introduction

In recent papers, Diamond and Dybvig (1983) and Bryant (1980) develop models of banking to explore bank runs and their prevention.[1] Although the focus of these papers is bank runs, both identify the demand deposit as a mechanism that facilitates risk sharing. Bryant (1980) characterizes the demand deposit as providing "insurance" for an otherwise uninsurable event. Diamond and Dybvig (p. 402) state that "banks issuing demand deposits can improve on a competitive market by providing better risk sharing. . . ." These authors raise an important issue—the role of demand deposits in risk sharing. But, given they are primarily concerned with modeling bank runs and deposit insurance, they do not fully explore this very important issue.

The purpose of this paper is to explore in depth the role of the demand deposit in risk sharing. Diamond and Dybvig demonstrate in their model that demand deposits allow greater risk sharing than direct investment in the underlying technology. For their model, I show that dividend-paying equity shares provide the same risk-sharing opportunities as demand deposits but do not introduce the possibility of a bank run. This result only requires that a market exist for the ex-dividend shares. I also show that equity shares cannot be used to achieve the same allocation as demand deposits for a large

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class of economies. The fact that they can in Diamond and Dybvig's environment follows directly from the extreme nature of their assumed preference structure. I show for fairly general preference structures that demand deposits provide opportunities for risk sharing beyond those provided by equity shares—whether the firm issuing the shares pays dividends or not.

Trading restrictions are, however, an essential element of the story. Demand deposits facilitate risk sharing because markets are incomplete. Hart (1975) demonstrates that, in a sequence of market settings, adding a spot market at a point in time may not even weakly increase the space of (explicitly or implicitly) marketed claims and thus may make everyone worse off. The economy modeled here provides an example of this phenomenon. The risk-sharing opportunities provided by demand deposits disappear if certain trading opportunities are introduced. The demand deposit allows risk sharing because it is a mechanism by which individuals of different types each self-select an allocation that is designated for that individual's type. For self-selection to take place, the alternative allocations must be incentive compatible. Additional trading opportunities may eliminate potential risk sharing because, after the introduction of the new trading possibilities, the alternative allocations may no longer be incentive compatible. If they are not, individuals of all types do not self-select and the mechanism breaks down.

In Section 2, I review Diamond and Dybvig (1983) and demonstrate that trading in the equity shares of a dividend-paying firm results in the same allocation achieved using demand deposits. In Section 3, I modify the preference structure assumed by Diamond and Dybvig and show that in this case the use of demand deposits allows greater risk sharing than does trading in the shares of a firm. The role of trading restrictions is developed in Section 4. I demonstrate that the equilibria in the economies discussed in sections 2 and 3 no longer exist if trading restrictions are relaxed or if trading in a new asset is introduced. Section 5 contains conclusions and suggestions for future research.

2. Risk Sharing Using Equity Shares in the Diamond/Dybvig Model

Diamond and Dybvig (1983) show in their model that demand deposits improve on the risk sharing achieved in a competitive market for the underlying technology. In this section, I show that trading in the shares of a dividend-paying firm achieves the same allocation as demand deposits—without incurring the risk of a bank run.

To begin, let us review their model and results. The main features are discussed below.

Production Technology

All investment in the production of the single good in the economy occurs at $T = 0$ (designate time as $T = 0,1,2$). The production process is infinitely divisible. Any portion of the production can be interrupted at $T = 1$ and immediately yield a total return equal to the initial investment, but then no additional return occurs in period 2. Otherwise, if the production process is not interrupted at $T = 1$, at $T = 2$ it yields a total return per unit invested of $R > 1$. R is a constant known to all at $T = 0$.

Individuals can store the good at no cost. This storage is not publicly observable.

Preference Structure

There is a continuum of individuals of total measure one. At $T = 0$, all individuals are identical. Their types are unknown to themselves as well as others. Types are independently and identically distributed. A person is of type 1 with probability t and of type 2 with probability $1 - t$.[2] At $T = 1$, each individual learns his own type but learns nothing regarding the types of others. Either he is of type 1 and dies at the end of period 1 or he is of type 2 and lives through the end of period 2. Furthermore, preference for consumption in periods 1 and 2 (where c_T indicates consumption in period T) is assumed to have the utility representation

$$\mu(c_1,c_2,\theta) = \theta u(c_1) + (1 - \theta)\rho u(c_1 + c_2)$$

where θ is a random variable equal to 1 if the individual is of type 1 and 0 if the individual is of type 2 and where $1 \geq \rho > 1/R$ and $u:R_{++} \rightarrow R$ is twice continuously differentiable, increasing, strictly concave, satisfies the Inada conditions $u'(0) = \infty$ and $u'(\infty) = 0$, and has relative risk aversion $cu''(c)/u'(c) > 1$ everywhere.

In addition, it is assumed that type is unobservable and that there exists no credible way of revealing one's type. That is, traditional insurance contracts tied to the observation of one's type are ruled out.

At $T = 0$, individuals are assumed to maximize the expectation of their state-dependent utility function $\mu(c_1,c_2,\theta)$ where c_{ik} represents consumption in period i for an individual of type k and where

$$E_\theta(\mu(c_1,c_2,\theta)) = tu(c_{11}) + (1 - t)\rho u(c_{12} + c_{22})$$

since $E_\theta(\theta) \equiv t$.

Endowments

Each consumer receives an endowment of one unit of the good in period 0 (and none at any other time).

Demand Deposit Contracts

A *demand deposit* is defined as a contract that requires an initial investment at $T = 0$ in exchange for the right to withdraw per unit of investment (at the discretion of the depositor and conditional on the bank's solvency) either r_1 in period 1 or $R(1 - r_1 f)/(1 - f)$ in period 2, where R is the total return on the two-period technology and f is the proportion of individuals who withdraw in period 1. Of course, the depositor who attempts to withdraw after the bank runs out of funds gets nothing. If r_1 is greater than 1, then the demand deposit provides insurance against being of type 1 since in that case the type 1 individuals get some benefit from the two-period production process even though they die before the process comes to fruition. However, if that is the case, there is the potential for a bank run since if more than $1/r_1$ of the individuals choose to withdraw their funds in the first period the bank runs out of funds.

Given this specification, Diamond and Dybvig show that using demand deposit contracts leads to two possible Nash equilibria. The first of these achieves socially optimal risk sharing. The other equilibrium is interpreted as a bank run. All depositors wish to withdraw all of their funds from the bank in the first period. Unfortunately, given the demand deposit contract, the bank does not have sufficient funds to meet the withdrawal demand, and consequently the bank fails. Type 1 depositors who are able to withdraw their funds before the bank fails receive their optimal allocation, and type 2 depositors who are able to withdraw their funds before the bank fails receive an amount less than their optimal allocation. Remaining type 1 and type 2 depositors, who do not withdraw before the bank fails, receive nothing.

Socially Optimal Risk Sharing

The socially optimal allocation is found by solving the following constrained optimization:

$$\max_{c_{11},c_{12},c_{21},c_{22}} \quad tu(c_{11}) + (1 - t)\rho u(c_{12} + c_{22})$$

subject to

$$(c_{11} + c_{21}/R)t + (c_{12} + c_{22}/R)(1 - t) = 1$$

and

$$c_{ik} \geq 0$$

for $i = 1, 2$ and $k = 1, 2$.[3] If demand deposits are to offer a social improvement, individuals must not be able to achieve their optimal allocation by directly investing in the production technology. Directly investing, an individual can be assured of the choice between either one unit of the good in period 1 or R units of the good in period 2. Therefore, if demand deposits are to have a role in the economy, the social optimum must be an improvement over this allocation. To assure that this is the case, Diamond and Dybvig assume that the coefficient or relative risk aversion is greater than 1 everywhere. Given this assumption, the social optimum is characterized by:

$$c_{12}^* = c_{21}^* = 0$$

$$c_{11}^* > 1$$

$$c_{22}^* = R(1 - c_{11}^* t)/(1 - t) \qquad (1)$$

where the * indicates optimality and $c_{22}^* > c_{11}^*$.

Review and Discussion of Results

Diamond and Dybvig show that by setting $r_1 = c_{11}^*$, the socially optimal allocation can be achieved as the superior symmetric Nash equilibrium using demand deposits. This equilibrium allocation results if the late diers conjecture that only the early diers will withdraw their funds in period 1. If late diers believe that everyone else will withdraw their funds in the first period, then they also want to withdraw theirs, and the bad equilibrium—a bank run—obtains. They also show that giving the bank the ability to suspend the convertibility of deposits ensures that only the good equilibrium allocation is a Nash equilibrium outcome.

Optimal Risk Sharing Using Equity Shares

Consider a firm that has access to the two-period technology and raises an amount of capital, C, by issuing shares (with a price of 1) at $T = 0$. Having purchased the shares of the firms, at $T = 0$ the shareholders also decide the firm's production policy and declare a per-share dividend, D, payable at $T = 1$ to the shareholder of record at $T = 0$. Thus, each share represents the right to a dividend of D at $T = 1$ and liquidating dividend of $R(C - D)$ at $T = 2$.

At $T = 1$, the shareholders receive their dividends and a market in the ex-dividend shares opens. All individuals now know their own type, so there are potential gains from trade. The type 1 individuals want to trade their ex-dividend shares for additional period 1 consumption goods. The type 2 individuals are indifferent between period 1 and period 2 consumption. Thus, if the price of ex-dividend shares is less than $R(C - D)$, they are willing to trade the period 1 consumption goods they received as a dividend for

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additional ex-dividend shares [which represent the right to $R(C - D)$ units of period 2 consumption].

Suppose that, instead of investing in demand deposits at $T = 0$, all individuals invest in this firm (i.e., $C = 1$). If the per-share dividend, D, is set at tr_1 and at $T = 1$ trading takes place in the ex-dividend shares, the social optimum obtains.

The price of the ex-dividend shares is determined by market clearing and is $r_1(1 - t)$. By (1), we know that $R(C - D) = R(1 - tr_1) > r_1(1 - t)$. Therefore, the type 2 individuals are willing to trade at this price. The fraction t of the total population who are of type 1 will trade their rights to $tR(1 - tr_1)$ in liquidating dividends for $(1 - t)r_1t$ in current consumption. Thus, each type 1 individual nets tr_1 in dividends plus $((1 - t)r_1t)/t = (1 - t)r_1$ from the sale of ex-dividend shares, giving each type 1 a total of r_1 in period 1. Each type 2 individual receives $R(1 - tr_1)$ in liquidating dividends plus $(tR(1 - tr_1))/(1 - t)$ in additional liquidating dividends from his ex-dividend share purchase, netting each type 2 individual $(R(1 - tr_1))/(1 - t)$ in period 2 consumption. This allocation is, of course, the social optimum.

One question remains. Will the firm pay a dividend of tr_1 at $T = 1$? The answer is yes. At $T = 0$, all the shareholders are identical. Thus, the social optimum coincides with each individual's optimum. Therefore, at $T = 0$ the shareholders unanimously agree to declare a dividend of tr_1 payable at $T = 1$.[4] At $T = 1$, individuals know their types and no longer agree on the dividend policy. However, having declared the dividend at $T = 0$, at $T = 1$ the firm has no choice but to pay the dividend.

This analysis requires that individuals can only invest in the shares of the firm. Similarly, Diamond and Dybvig must assume that individuals can only invest in demand deposits. In Section 4, I show that introducing additional investment and trading opportunities can eliminate the equilibria in both cases and thus reduce risk sharing.

3. Improving Risk Sharing with Demand Deposits

In the last section I showed that, in the context of Diamond and Dybvig's model, demand deposits cannot improve on the allocations achieved with a competitive market for a dividend-paying firm. This is not generally true. In this section I show that, with a different, yet quite reasonable, preferable structure, demand deposits do improve on competitive markets—even those with dividend-paying firms.

Let us consider a modification of the economy modeled by Diamond and Dybvig. This new economy retains the same initial endowment and two-period production technology but has a different preference structure and a revised definition of a demand deposit.

Smooth Preferences

Assume that all individuals live in both periods and have preferences that are smooth in period 1 and period 2 consumption. Types now reflect differences in time preferences. Type 1 individuals prefer more consumption in the first period than do type 2 individuals. Types are still independent, and the ex ante probabilities of being type 1 and type 2 are still t and $1 - t$, respectively.

Formally, preferences have the utility representation

$$\mu(c_1,c_2,\theta) = \theta U(c_{11},c_{21}) + (1 - \theta)V(c_{12},c_{22})$$

where θ is an indicator variable for type 1 and $U(.,.)$ and $V(.,.)$ are twice continuously differentiable, increasing, strictly concave, and satisfy the Inada conditions. Further, let

$$U_1(c_1,c_2)/U_2(c_1,c_2) > V_1(c_1,c_2)/V_2(c_1,c_2)$$

for all values of (c_1,c_2).

Type is still unobservable, and there is still no credible way of revealing one's type. Consequently, there is no opportunity to construct traditional insurance contracts against being of a particular type.

At $T = 0$, individuals are assumed to maximize the expectation of their state-dependent utility function $\mu(c_1,c_2,\theta)$, where

$$E_\theta[\mu(c_1,c_2,\theta)] = tU(c_1,c_2) + (1 - t)V(c_1,c_2)$$

since $E_\theta(\theta) \equiv t$.

Demand Deposits

Since we are now concerned with the potential for risk sharing using demand deposits and not with modeling bank runs, we use a definition of demand deposits that assumes that bank runs are avoided somehow. The definition would need to be expanded if this possibility were allowed.

Define a demand deposit as a contract that requires an initial investment at $T = 0$ in exchange for the right to withdraw per unit of investment (at the discretion of the depositor and conditional on the bank's solvency) either:

$$x_1 \text{ in period 1 and}$$

$$x_2 \text{ in period 2}$$

or

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As part of the definition, assume that trading in demand deposits is prohibited. The last assumption is discussed in Section 4.

At first glance, this contact bears little resemblance to the usual definition of a demand deposit. However, one's intuition may be improved if the contract is viewed as implicitly representing two financial instruments. The first is a direct investment in the two-period technology, and the second is a contract quite similar to a demand deposit. To see this, consider the following: in this economy, regardless of type, optimally there is a minimal amount everyone would consume in the second period. At $T = 0$, that amount can be assured for the second period by a direct investment in the two-period technology. Having assured themselves of this second-period income, type 1 individuals will want to consume all of their remaining wealth in the first period. On the other hand, type 2 individuals will want to consume a portion of their remaining wealth in each of the two periods. Since types are not known at $T = 0$, a contract that allows investors the option of taking all their return at $T = 1$ or taking a portion of their return in each period can facilitate risk sharing. The second instrument represented in the definition of a demand deposit does exactly this.

So, the demand deposit defined here represents two contracts. The first requires an investment of x_2/R at $T = 0$ and guarantees a return of x_2 at $T = 2$. The second requires an investment of $1 - x_2/R$ at $T = 0$ in exchange for the right to either

x_1 in period 1 and

0 in period 2

or

y_1 in period 1 and

$y_2 - x_2$ in period 2.

The second contract is similar to the usual definition of a demand deposit. Furthermore, if $1 - x_2/R < x_1$, a bank run could potentially arise with this second contract.

Demand Deposits vs. Trading in Equity Shares

To compare the extent to which demand deposits and trading in equity shares allow risk sharing, we must compare the allocations that can be obtained using the two mechanisms. Simultaneously, we can investigate whether the allocations obtained are efficient. My approach is to characterize an upper bound and then demonstrate the conditions under which the bound can be achieved by each mechanism.

At $T = 0$, all individuals are essentially identical. Therefore, subject only to the resource constraint, the ex ante efficient allocation is found by solving the following maximization:

$$\max_{c_{11},c_{12},c_{21},c_{22}} \quad tU(c_{11},c_{21}) + (1-t)V(c_{12},c_{22})$$

subject to

$$(c_{11} + c_{21}/R)t + (c_{12} + c_{22}/R)(1-t) = 1.$$

The assumed Inada conditions guarantee an interior solution to this problem, and smoothness guarantees that the solution is unique. The optimal allocation satisfies the budget constraint and the following first-order conditions:

$$U_1 = V_1,$$

$$U_2 = V_2, \text{ and}$$

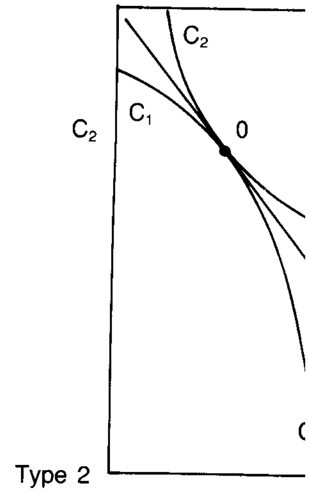
$$U_1/U_2 = R.$$

To simplify the analysis, assume that individuals cannot store the good. Allowing private storage complicates the analysis without changing the general nature of the results.

The optimal allocation gives all individuals of the same type identical two-period consumption streams. Further, there is an implicit aggregate production decision made in the optimization. If the total endowment is invested in the two-period production process at $T = 0$, the period 1 consumption must be obtained through the liquidation of a portion of the original investment. Given this implicit decision, we can characterize the social optimum as a point in an Edgeworth's box.

Consider figure II-1. The dimensions of the Edgeworth's box are determined by the aggregate production choice. The axes represent consumption in the first and second periods. The lower left-hand corner is the origin from the perspective of type 2 individuals, and the upper right-hand corner is the origin from the perspective of type 1 individuals. The curves C_1C_1' and C_2C_2' are indifference curves for type 1 and type 2 individuals, respectively. The optimal allocation is represented in figure II-1 as point 0. Notice that this is a point of tangency for the two indifference curves. From the first-order conditions we know that the slope of the indifference curves at point 0 is $-R$.

The use of the Edgeworth's box is helpful because it allows us easily to identify three alternative characterizations, represented by figures II-1 through II-3, of the social optimum. I now discuss these three characterizations.



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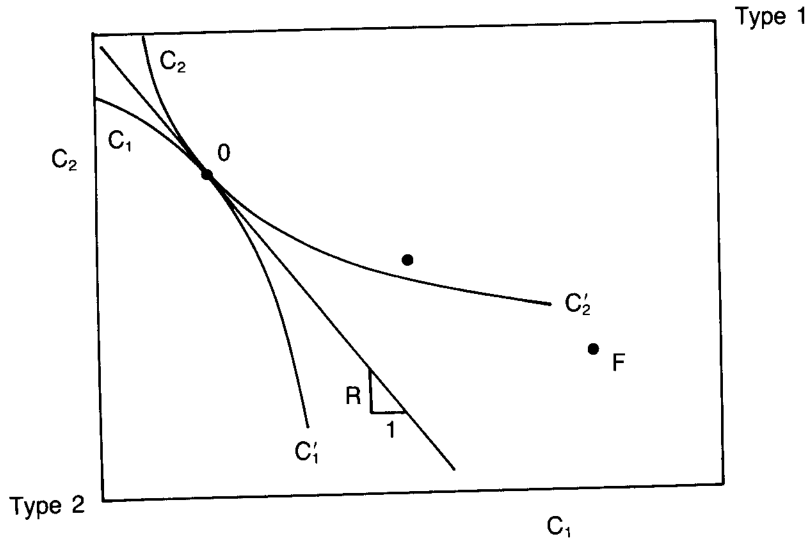


Figure II-1

Two properties of the social optimum are of interest. The first is whether the social optimum represents an incentive-compatible allocation. An allocation is incentive compatible if individuals of each type prefer their own allotment to the allotment of the other type. That is, the social optimum represents an incentive-compatible allocation if

$$U(c_{11}^*, c_{21}^*) \geq U(c_{12}^*, c_{22}^*)$$

and

$$V(c_{12}^*, c_{22}^*) \geq V(c_{11}^*, c_{21}^*)$$

where the * indicates optimality.[5]

In figure II-1, the point 0 represents an incentive-compatible allocation. This is seen by noting that an allocation that gives individuals of each type the other type's allocation, represented by point *F*, moves neither type into their preferred set. On the other hand, in figure II-2 the point 0 represents an allocation that is not incentive compatible. In that figure, reversing the allocations moves the type 2 individuals into their preferred set.

The second property of the optimal allocation that is of interest is whether the line tangent to the two indifference curves at the social optimum passes through the identical endowment point. In figure II-3, the tangent line does pass through this point (designated *E*). The identical endowment point represents an allocation that allots individuals of both types the same two-period

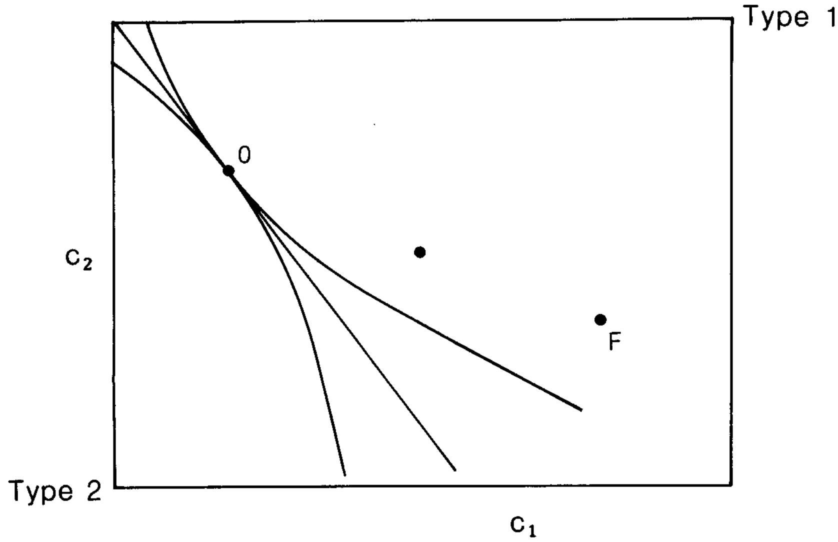


Figure II-2

endowment (as happens with equity shares). That is, type 1 individuals receive the same amount of endowment in each period as do type 2 individuals. Note, however, that the amount received may vary from period 1 to period 2. We shall see that, if the social optimum has this property, both demand deposits and equity shares can be used to achieve the optimal allocation.

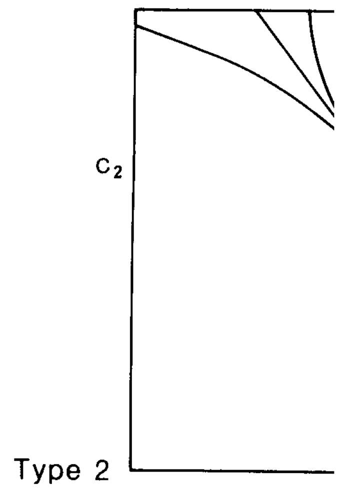
First, let us consider incentive compatibility, which is of interest because types are not observable. If demand deposits are to be used in achieving the optimal allocation, individuals of different types must self-select the allotment that is designated for their type. If the optimal allocation is not incentive compatible, individuals do not self-select appropriately. Thus, we have the following theorem:

THEOREM 1. *Assume that demand deposits cannot be traded. Then, demand deposits can be used to achieve the social optimum subject only to the resource constraint if and only if this optimum is incentive compatible.*

PROOF. Let $(c^*_{11},c^*_{21},c^*_{12},c^*_{22})$ be the social optimum. If this allocation is incentive compatible, then it can be achieved using demand deposits by setting the terms of the demand deposit so that

$$x_1 = c^*_{11},$$

$$x_2 = c^*_{21},$$



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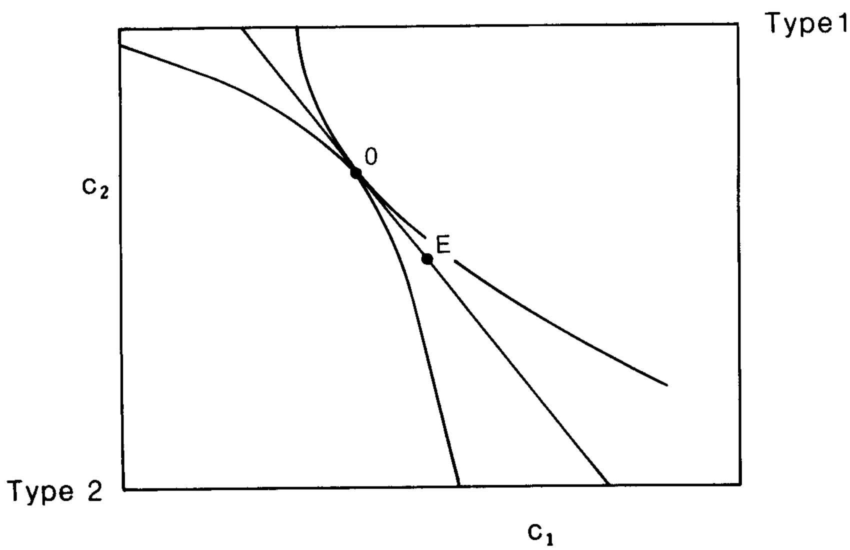

Figure II-3

$$y_1 = c_{12}^*, \text{ and}$$

$$y_2 = c_{22}^*.$$

Since demand deposits cannot be traded and the good cannot be stored, an individual can only choose between two consumption streams, (c_{11}^*, c_{21}^*) and (c_{12}^*, c_{22}^*). If the social optimum is an incentive compatible allocation, type 1 individuals prefer the first allotment and type 2 individuals prefer the second. Thus, individuals self-select appropriately and the social optimum is achieved.

On the other hand, if the social optimum is not incentive compatible, both types prefer the same allotment. Ex ante optimality requires that individuals of one type take an allotment that is dominated. Types are not observable, so there is no way to force individuals of that type to select the dominated allotment. Thus, in this case the ex ante optimum cannot be achieved using the demand deposits. Q.E.D.

Notice that I ignore the possibility of bank runs in the case where the social optimum is incentive compatible, because the focus of this work is on potential risk-sharing opportunities. Bank runs can be avoided in this case by using suspension of convertibility as in Diamond and Dybvig (1983).

Theorem 1 characterizes the allocations that are achievable using demand deposits, the key factor being whether the allocation is incentive compatible. If trading in equity shares is used instead, the key factor becomes whether the tangent line at the social optimum passes through the identical endowmand deposits can generally achieve greater risk sharing than equity shares traded in competitive markets since, in the demand deposit economy, the same objective function is being maximized over a less constrained set.

The necessity of restricting trade in the demand deposit economy, which is also apparent from this interpretation, can be seen by contrasting this economy with the equity one. At $T = 1$ in the equity economy, all individuals have the right to identical two-period dividend streams. Having learned their types, they trade from these identical streams. Clearly, every individual has the same amount of wealth at $T = 1$. In the deposit economy, on the other hand, no trade is allowed. Thus, each individual must consume whichever withdrawal stream that person chooses. The two possible withdrawal streams are constructed to be incentive compatible—that is, all those persons of type 1 prefer one stream whereas all those in type 2 prefer the other, given that they cannot trade ex post. Since no ex post trade is allowed, these two withdrawal streams do not necessarily represent the same amount of wealth where wealth is defined as the value in a world where ex post trade is allowed. So, using the deposit contract can accomplish potentially greater risk sharing than using the equity contract because allocations are not restricted to being of equal wealth across types. For example, individuals could be insured against being type 1 to a greater extent (than in an equity world) with a deposit contract that offered two incentive-compatible withdrawal streams (given no ex post trade) where the stream preferred by type 1 individuals represented more wealth than the stream preferred by those in type 2.

Note that such insurance never takes place in the environment analyzed by Diamond and Dybvig. Even if trading in demand deposits is prohibited, demand deposits do not improve on a competitive market in a dividend-paying firm because, in their model, the social optimum is always a competitive equilibrium from identical endowments. The social optimum has this property because of the assumed preference structure. In their model, type 1 individuals only have utility for consumption in period 1. Hence, their marginal rate of substitution of period 2 consumption for period 1 consumption will be infinite. On the other hand, consumption in period 1 vs. that in period 2 is a perfect substitute for type 2 individuals. Therefore, the marginal rates of substitution for the two types are not equalized at the social optimum. This preference structure allows the competitive price of period 2 consumption in terms of period 1 consumption to be any number between 0 and 1. The competitive price in this range is determined solely by the ratio of the aggregate endowments in periods 1 and 2. As a result, the social optimum can always be achieved as a competitive equilibrium from identical endowments.

The above discussion should also clarify the nature of the trading restric-

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subject to

$$(c_{11} + c_{21}/R)t + (c_{12} + c_{22}/R$$

and

$$V(c_{12},c_2$$

Restricting attention to nonstoc
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THEOREM 3. *If demand deposits*
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PROOF. (Demand deposits.) Let
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for U and V, we know that $\hat{c}_{ik} > 0$
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fore, there are potential gains from
the constrained optimum is a comp

tions imposed in the deposit economy. Not only is trade in the deposit contract prohibited, but also any type of trade or credit market that effectively circumvents this trading restriction. For example, a one-period credit market at $T = 1$ (using demand deposits as collateral) is ruled out. The key is not actually the total prohibition of such credit markets—rather, it is required that such markets be sufficiently inefficient to preclude effective circumvention of the restriction on the trading of deposits.

I have shown that neither mechanism considered achieves perfect risk-sharing in this economy unless the optimal allocation is incentive compatible. Now consider how these mechanisms compare when the optimal allocation is not incentive compatible. In the problem at hand, this means we wish to perform the following optimization:

$$\max_{c_{11},c_{12},c_{21},c_{22}} \quad tU(c_{11},c_{21}) + (1-t)V(c_{12},c_{22}) \tag{3}$$

subject to

$$(c_{11} + c_{21}/R)t + (c_{12} + c_{22}/R)(1-t) = 1;\ U(c_{11},c_{21}) \geq U(c_{12},c_{22})$$

and

$$V(c_{12},c_{22}) \geq V(c_{11},c_{21}).$$

Restricting attention to nonstochastic allocations, we see that the following theorem distinguishes between the risk-sharing properties of demand deposits and equity shares when the social optimum is not incentive compatible.[7]

THEOREM 3. *If demand deposits cannot be traded and the social optimum subject only to the resource constraint is not incentive compatible, demand deposits can be used to achieve the optimal nonstochastic, incentive-efficient allocation, but equity shares cannot.*

PROOF. (Demand deposits.) Let $(\hat{c}_{11},\hat{c}_{21},\hat{c}_{12},\hat{c}_{22})$ be the solution to (3) with the choice variables restricted to being nonstochastic. This allocation is incentive compatible by construction. Therefore, by an argument similar to that of Theorem 1, this allocation can be achieved through the use of demand deposits.

(Equity shares.) We have seen that equity shares can only achieve allocations that are competitive equilibria from identical endowments. Suppose $(\hat{c}_{11},\hat{c}_{21},\hat{c}_{12},\hat{c}_{22})$ is a competitive equilibrium. Since the Inada conditions hold for U and V, we know that $\hat{c}_{ik} > 0$ for $i = 1, 2$ and $k = 1, 2$. If one of the incentive compatibility constraints is binding, at the constrained maximum the marginal rates of substitution of the two types are not equalized. Therefore, there are potential gains from trade. This contradicts the premise that the constrained optimum is a competitive equilibrium. Q.E.D.

4. The Need for Trading Restrictions

Thus far, I have assumed that demand deposits cannot be traded. Furthermore, I have stated that adding assets may introduce problems in the economies examined. In this section I show that if demand deposits can be traded, optimal risk sharing is guaranteed in neither Diamond and Dybvig's economy nor the economy with smooth preferences described in Section 3. This breakdown in risk sharing happens because the social optimum is no longer a Nash equilibrium if individuals have additional trading opportunities.

In the Diamond and Dybvig Model

The equilibrium in the Diamond and Dybvig model has the following flaw: if ex post trading is possible and new assets can be introduced, on the margin individuals have no incentive either to invest in the shares of a dividend-paying firm or to deposit their funds in the bank. Consider an individual who either invests directly or in a marginal firm or bank (sufficiently small so as not to affect prices in equilibrium) that offers only the right to R units of the good at $T = 2$ for each unit of investment at $T = 0$. Assuming that prices do not change, an individual is better off investing in this "deviant" firm or bank. If at $T = 1$ the individual is of type 2, he or she can just hold onto the shares or withdrawal rights and receive $R > (R(1 - tR_1))/(1 - t)$ at $T = 2$. On the other hand, if the individual is of type 1, the shares or withdrawal rights can be sold for $((1 - t)r_1)/(1 - tr_1) > r_1$.[8] In either case, the person is better off.

This unraveling problem is a result of the preference assumptions of Diamond and Dybvig. Since type 1 individuals have no utility for period 2 consumption, marginal rates of substitution are not equalized across types at the social optimum. Failure to equalize marginal rates of substitution allows the unraveling problem to exist. Diamond and Dybvig do not address this point; presumably, they assume it away implicitly. This flaw results from marginal rates of substitution not being equalized at the social optimum.

In an Economy with Smooth Preferences

If preferences are smooth and demand deposits can be traded costlessly, the only allocations that can be achieved are competitive equilibria from equal endowments. This assertion holds regardless of whether equity contracts or demand deposits are used in the economy. In this subsection, this assertion is proved and an example illustrating the theorem is provided.

For the remainder of the section, let preferences be as described in Section 3—smooth and representable by functions that satisfy the Inada conditions in both arguments. These assumptions are sufficient to assure that

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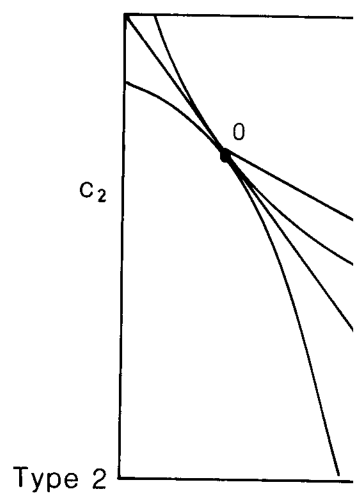

the social optimization has an interior solution and that at the social optimum the marginal rates of substitution are equalized across types. Having made these assumptions, consider figure II-5, which is an Edgeworth's box representation of the socially optimal allocation between type 1 and type 2 individuals. First notice that the social optimum, represented by point 0, is individually incentive compatible. This is seen by noting that an allocation that gives each type the other type's optimal allocation, represented by point F, moves neither type into that individual's preferred set. However, with ex post trading, individual incentive compatibility is not the appropriate incentive compatibility concept. For example, a group of individuals could form a coalition, have some individuals select the inappropriate withdrawal strategy, redistribute their total allocation, and all be better off. In particular, for the situation depicted in figure II-5, a group of type 2 individuals could form a coalition and, having agreed to divide the total allocation equally, each achieve any point along the line $0F$. Part of this line falls in their preferred set. Thus, all the individuals in the coalition can be made better off than they were at point 0. Unless they are restricted from forming coalitions, coalition incentive compatibility is the appropriate concept of incentive compatibility in this situation.

Note, however, that the set of all coalitionally incentive compatible points is defined to be the core of the economy. Furthermore, Aumann (1964)

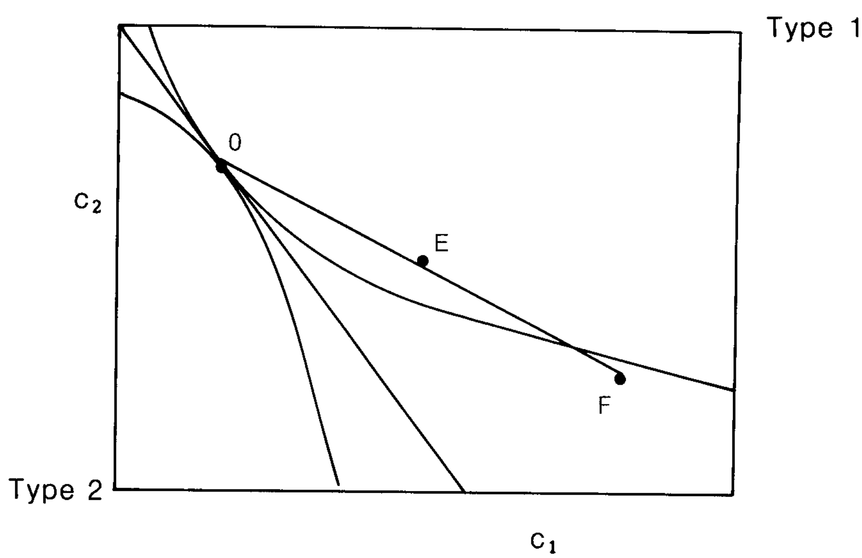

Figure II-5

proves that, in an economy with a finite number of types (for which there exists a continuum of individuals of each type), the core of the economy collapses to the set of competitive equilibria. Thus, the formation of coalitions can be thought of as occurring through the process of trading. The following theorem shows that allowing trade in demand deposits makes them equivalent to equity shares.

THEOREM 4. *If demand deposits can be traded costlessly, equity shares and demand deposits are equivalent risk-sharing mechanisms.*

PROOF. First, it is easy to see that the use of deposit contracts can achieve any allocation that can be achieved using equity contracts. The deposit contract just needs to specify the two alternative withdrawal streams to be final allocations achieved in the equity economy for individuals of type 1 and type 2. Since these allocations represent a competitive equilibrium, there are no gains from further trade if everyone self-selects appropriately. Furthermore, neither type has any incentive to misrepresent its type since both withdrawal streams represent the same amount of wealth and thus could be used to achieve the same set of possible allocations through trade.

Now, it is shown that if trade in demand deposits is allowed, then the best allocation that can be achieved using demand deposits is the allocation achieved using equity shares. The possibility of ex post trade affects each type's choice of withdrawal stream. To be consistent with the optimal liquidation decision, we know that the one-period gross return from holding demand deposits from time 1 to time 2 must be R. Thus, being able to compute what prices will be in the market at $T = 1$, everyone will prefer the allocation that offers the greatest wealth. This constrains the bank to offering withdrawal streams representing the same amount of wealth. Given that each withdrawal stream offers the same amount of wealth, either stream could be achieved through trade in the equity economy. Thus, any allocation achieved in the deposit economy can also be achieved in the equity economy. Q.E.D.

The following example demonstrates how the deposit equilibrium breaks down if coalitions can form costlessly (or if frictionless trade takes place).

EXAMPLE. Let preferences be smooth and trading be costless and unrestricted. Assume that each individual believes all other individuals act myopically (i.e., they do not consider potential trades in choosing their withdrawal strategy). Then, unless the optimal allocation is a competitive equilibrium from equal endowments, all the individuals of one of the two types will find it in their best interest to choose the inappropriate myopic allocation. They will do so by believing that they will always be able to achieve a Pareto-improving allocation by making a finite number of uncon-

ditional trades with any arbitr
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Consider figure II-6. A de
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Notice that the tangent lin
Therefore, by the smoothness
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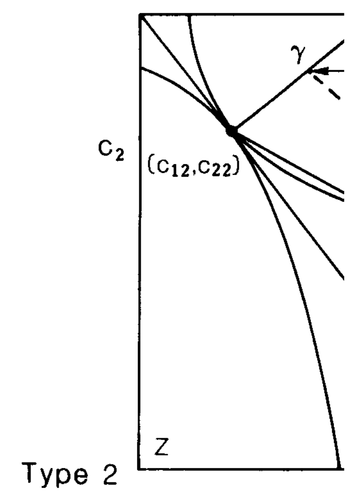

ditional trades with any arbitrary group of individuals (i.e., the type mix of the group does not matter).

Consider figure II-6. A deviant type 2 individual wishes to trade from point β to point γ. Note that, using point Z as the origin, β has coordinates (c_{11},c_{21}) and γ has coordinates $(c_{11} - a, c_{21} + Ma)$. Thus, the deviant type 2 individual wishes to trade a units of first period consumption for Ma units of second-period consumption.

Notice that the tangent line at point a has slope $-R$ and that $R > M$. Therefore, by the smoothness of preferences, we know that there exists δ_1, $\delta_2 > 0$ such that

$$(c_{11} + \delta_1, c_{21} - M\delta_1) \succ_1 (c_{11}, c_{21})$$

and

$$(c_{12} + \delta_2, c_{22} - M\delta_2) \succ_2 (c_{12}, c_{22})$$

where $\succ_i$ indicates strict preference for type i. Now, let δ be defined to be $\min(\delta_1,\delta_2)$. It is clear that the deviant individual, to succeed in moving from point β to point γ, need only make at most $a/\delta < \infty$ Pareto-improving trades with individuals of either type who have chosen their myopic allocation.

Theorem 4 raises severe doubts as to the viability of demand deposits in a frictionless economy in which all individuals have smooth preferences. Equity contracts could always be used to obtain the same equilibria. For

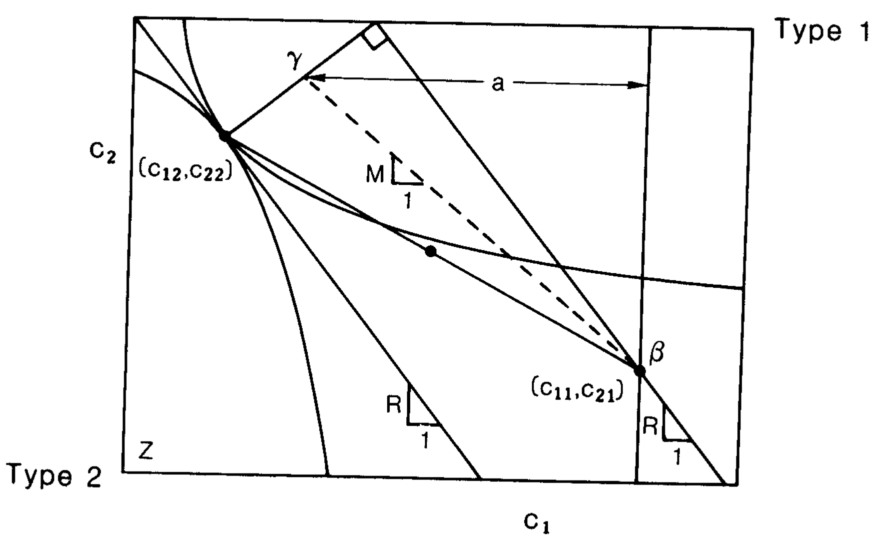

Figure II-6

demand deposits to have an important social role, trade in deposits must be prohibited or costly. This includes the prohibition of a frictionless credit market at time 1, since such a market effectively circumvents any restrictions on the trading of demand deposits. Note, however, that at $T = 0$ everyone in the economy would agree to such restrictions because they allow all individuals to improve their ex ante expected utility.

5. Conclusions

This paper has attempted to clarify and add support to the social role for demand deposits identified by Bryant (1980) and Diamond and Dybvig (1983). That trading restrictions are necessary for the existence of a social role for demand deposits in models of the type examined here is a principal contribution of this paper. This result is similar to the result in the taxation literature (Hammond 1979, 1983), which shows that whenever the optimal taxation policy is nonlinear it is vulnerable to resale. Whenever frictionless markets exist, individual incentive compatibility of a mechanism is not sufficient to guarantee truthful revelation of types.

Notes

1. I would like to express my appreciation to Sudipto Bhattacharya for the many insights he provided in our discussions of this and related work. I would also like to thank Ben Bernanke, Milt Harris, Ed Prescott, Ed Robbins, Neil Wallace, and Bob Wilson for helpful comments on earlier drafts of this paper. All remaining errors are, of course, mine.

2. Although there are problems with the law of large numbers when there is a continuum of i.i.d. random variables, Feldman and Gilles (1985) and Judd (1985) have demonstrated that these problems can be alleviated by imposing sufficient regularity on the mathematical structure.

3. Social optimality is used interchangeably with ex ante expected utility maximization of a representative individual. Note that at $T = 0$ everyone in the economy is identical.

4. If we assume that the firm is valued as a discounted dividend stream and that the appropriate discount factor for the second period is R (the rate of transformation), this dividend policy is not necessarily value maximizing; nonetheless, it is unanimously agreed upon by all shareholders. This should not be surprising. Given the preference structure, individual's marginal rates of substitution are not set equal to the marginal rate of transformation in equilibrium. Thus, the (marginal) rate of transformation is not necessarily the appropriate discount factor.

5. If storage were allowed, the constraints would be

$$U(c_{11},c_{21}) \geq U(c_{12}(1 - S_1),c_{22} + S_1c_{12})$$

and

$$V(c_{12},c_{22}) \geq V(c_{11}(1 - S_2),c_{21} + S_2c_{11})$$

for all S_1 and S_2 between 0 and 1.

6. Note further that, given smooth preferences and the premises of Theorem 2, this dividend decision is also value maximizing (ex post).

7. The use of lotteries may offer
Prescott and Townsend (1984) for a
constrained allocations.

8. Recall from Section 2 that the
$T = 2$, is $r_1(1 - t)$. Therefore, the
$(1 - tr_1)$.

References

Aumann, R. J. 1964. "Markets with
Bryant, J. 1980. "A Model of Res
Banking and Finance 4:335–
Diamond, D. W., and P. H. Dybvig.
Journal of Political Economy
Feldman, M. and C. Gilles. 1985. "A
Uncertainty." *Journal of Eco*
Hammond, P. 1979. "Straightforwa
mies." *Review of Economic S*
———. 1983. "Multilateral Incentiv
per, Economics Department,
Hart, O. D. 1975. "On the Optimalit
nal of Economic Theory 10:41
Judd, K. L. 1985. "The 'Law of La
ables." *Journal of Economic*
Prescott, E. C., and R. M. Townsen
with Private Information." *Int*

7. The use of lotteries may offer improvements to the allocations considered here. See Prescott and Townsend (1984) for a discussion of the use of lotteries to improve incentive-constrained allocations.

8. Recall from Section 2 that the price of an ex-dividend share, which pays $R(1 - tr_1)$ at $T = 2$, is $r_1(1 - t)$. Therefore, the price of a share, which pays R at $T = 2$, is $(r_1(1 - t))/(1 - tr_1)$.

References

Aumann, R. J. 1964. "Markets with a Continuum of Traders." *Econometrica* 32:39–50.

Bryant, J. 1980. "A Model of Reserves, Bank Runs, and Deposit Insurance." *Journal of Banking and Finance* 4:335–44.

Diamond, D. W., and P. H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91:401–19.

Feldman, M. and C. Gilles. 1985. "An Expository Note on Individual Risk without Aggregate Uncertainty." *Journal of Economic Theory* 35:26–32.

Hammond, P. 1979. "Straightforward Individual Incentive Compatibility in Large Economies." *Review of Economic Studies* 46:263–82.

———. 1983. "Multilateral Incentive Compatibility in Continuum Economies." Working paper, Economics Department, Stanford University.

Hart, O. D. 1975. "On the Optimality of Equilibrium When Markets Are Incomplete." *Journal of Economic Theory* 10:418–43.

Judd, K. L. 1985. "The 'Law of Large Numbers' with a Continuum of IID Random Variables." *Journal of Economic Theory* 35:19–25.

Prescott, E. C., and R. M. Townsend. 1984. "General Competitive Analysis in an Economy with Private Information." *International Economic Review* 25:1–20.